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ABSTRACT

When left to their own devices, children are natural story tellers and they practice story telling in every aspect of their learning, including mathematics. The ways in which young children's mathematics learning is constrained distorts the potential for the kinds of experiences that can feed their narratives and help them to make sense of the narrative as well as shift them toward more conventional narratives. Teaching paradigmatic mathematics in the absence of either children's need to know or connections to their learning style is evidence of a need for changing the way mathematics is taught in the classroom. In addition, evidence suggests that research mathematicians use the very narrative strategies that are identified as productive for young children's learning. The current paradigm of learning mathematics fails to distinguish between knowing and knowledge and between the construction of narrative and its formalization and results in children finding mathematics unpalatable and unacceptable. An alternative narrative perspective on mathematics, consistent with approaches to the learning of other disciplines, enables the learner to inhabit and make sense of the mathematical world that they are entering. This approach treats early learning as a research practice in which story telling, the construction of convincing narratives, is key. Transcriptions of young children's social interaction and discourse provide evidence of children behaving in four ways which contribute to their mathematics learning: authoring, sense-making, collaborating, and using nonverbal narratives. Children's narratives can further provide information for teachers regarding their mathematical development. In classrooms where such information is valued and encouraged, and where learners are expected to reflect, and make and justify their inferences and deductions, the children freely identify their own understandings. (Contains 14 references.) (KB)

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Children's mathematical narratives as learning stories

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Introduction - Why does a narrative approach help children to learn mathematics?
Narrative, as I am sure we would all agree, is what we use to impose coherent meaning on experience. But narratives must not only be coherent to the narrator and the listener, or reader, they must also be connected, that is have a story to tell, and communicative. A narrative, I am claiming, engages narrator and listeners in an attempt to understand and explain the experience provoking it.

Narratives are personal to the narrator so that each account of an apparently shared experience will be different for each narrator. In mathematical terms, they will be similar but not congruent. We use narrative in order to pose, explore and respond to questions, to examine implications and to pursue and test conjectures against the substance of the experience. However, recounting a narrative requires a participatory audience, present or not. It is something we do in a community, rarely on our own.

A classroom constitutes a community of learners where narrative is often expected and recognised but rarely when mathematics is being taught or learned. So it falls to me to convince you that narrative is as appropriate to the learning of mathematics as it is to other kinds of learning. To do this, we have to share a number of stories.

The first story is about mathematics itself, the nature of the discipline. As I pointed out elsewhere (Burton, 1996):

> Despite a century of reinforcement for the notion of the Queen of Sciences as objective, universal, certain and infallible, ... recent work by sociologists, philosophers and historians makes the relationship of mathematics to its social and cultural (including political) roots more and more evident. Much of what is taught as immutable mathematics, especially at school level, is a social distillation of the results of refining strategies which have been particular to space and time. (p.31)

Mathematics, for me, then, is a socio-cultural artefact, like language or the arts. As such, it is available to the members of any community to revise, revisit, and play in order to improve its efficacy for them. Understood in this way, mathematics becomes a socio-cultural story told in a socially negotiable context. The fact that mathematics is shareable across many communities and cultures relates more to its utility and the development of its shared discourse than it does to how it is derived and the consequent meaning it carries..

So, next, we must turn to the classroom as a place where mathematical meaning is negotiated. As with all negotiation, this depends upon a number of factors, which are not present in the majority of current mathematics classrooms.

Etienne Wenger points out that "through the negotiation of meaning, it is the interplay of participation and reification that makes people and things what they are." (1998: 70). So participation, for Wenger is "a complex process that combines doing, talking, thinking, feeling, and belonging" (Ibid: 56). The classroom implications of this point to activity, discourse, and social interaction but also a recognition that participation, by giving meaning to our experiences, becomes "a constituent of our identities" (Ibid: 57).

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Reification, on the other hand, is "the process of giving form to our experience by producing objects that congeal this experience into 'thingness'" (Ibid: p.58) and "any community of practice produces abstractions, tools, symbols, stories, terms, and concepts that reify something of that practice in a congealed form" (Ibid: p.59). He sums up that "one advantage of viewing the negotiation of meaning as constituted by a dual process is that we can consider the various trade-offs involved in the complementarity of participation and reification" (Ibid: p.64). For me, one of these trade-offs is the recognition that the learning of mathematics is not simply a cognitive matter but involves people in their entirety.

To ensure that participation and reification operate in a balanced way requires that the classroom environment is supportive of both, separately, but also of how they interact together to produce negotiated meaning. Pupils have to be trusted to assume responsibility for themselves and their learning and to exercise that responsibility judiciously with respect to one another and to their teacher. The classroom has to be a place where there are stories to tell, so the work cannot be replicative; the pupils must be ready and willing to levy and respond to questions, have enquiry strategies to drive their activity and be reflective and convincing in presenting and arguing their summative stories. The kind of story-telling which underpins the learning of mathematics is not arbitrary. A teacher has to create the conditions to make the connections between the "imaginary" and the "paradigmatic", to use Bruner's (1986) terms (and see Burton, 1999). Instead of teaching mathematics as cognitively driven and unproblematically paradigmatic, using a narrative approach helps to locate the agency and the authorship of the learner in creating, arguing for and, ultimately, justifying the transformation of personal, or community, knowing into accepted public knowledge. "Preferred versions of knowledge are often consistent with Jerome Bruner's description of paradigmatic presentations. Nonetheless, they are narratives scribed by members of the community and offered as conforming to well-authenticated, acceptable conventions within a shared 'discourse community' (Resnick, 1991) with its own 'social language' (Bakhtin, 1986)." (Burton, 1999: 23)

Such an approach must expect some stories to conflict, and some pupils to pursue pathways not predicted by the teacher. Unlike a conventional classroom where "knowledge conflicts cannot emerge. A disagreement will always indicate that somebody has misunderstood something" (Skovsmose, 1993: p.175), a classroom in which meaning is expected to be negotiated, values disagreement as an opportunity to explore different understandings and search for communality.

Finally, Bruner (1986:25/6) lists three features of discourse which I see as being necessary to mathematical narrative. They are ***presupposition, subjectification*** and ***multiple perspective.*** Presupposition allows the creation of implicit meanings. Subjectification places at it's centre those who are undertaking the enquiry and, consequently, their questions, their strategies, their reflections. Multiple perspectives recognise, indeed celebrate, many views on the same activity or work.

To summarise, then, to accept a narrative approach to the learning of mathematics requires an acknowledgement of the agency and authorship of the learner. What are the features that such an acceptance imposes on an effective mathematics classroom?

They are:

Activity
Social interaction
Discourse - presupposition
- subjectification
- multiple perspectives

supported by a classroom environment which values the telling of stories as part of which pupils are expected to

Question
Challenge
Enquire
Reflect
Communicate

Now, let us turn to the narratives that, under these conditions, children can and do tell. I am using these narratives to draw attention to four aspects of what agentic children can do as they learn to learn mathematics. The four aspects are:

Authoring
Sense-making
Collaborating
Using non-verbal narratives

The transcriptions that I am presenting here are mainly taken from the thesis of one of my students, Gay Vaughan, who worked with pairs of 5-year-old children in order to map their zones of proximal development as, in different contexts, they attempted to come to know number. She was concerned that there appeared to be "a widespread belief in schools that the National Curriculum levels provided an effective linear hierarchical framework for planning children's numerical development" (Vaughan, 1999: 26) whereas she felt that an assessment model which sought to map zones of proximal development would be more likely to uncover richer learning. In terms of the features already listed which I claim to be necessary to narrative functioning as an effective way of learning mathematics, Gay was careful to set up activities which provided a context in which the children wanted to use, and took agentic control of their use of number. These included a dartboard where one child moved the dart in the outer circle, while the other had to say what the dart would score if it landed in a particular position; Roamer, an electronic toy which can be programmed to move; a measuring tape to take wrist, head and waist measurements; baby's feeding bottles labeled differently; shopping coupons, and other similar contexts. The assumption was that contexts would be meaningful for different children but that the variety would be sufficient for the data provided by each child to be meaningful. Gay worked with pairs of children, each of which was matched with a similar pair who did not have the experience of working with Gay so that assessment information could be compared. The exchanges were dialogic with Gay introducing into a conversation a question or an activity and inviting the children to respond and comment. For example, in her first interview she chatted with the children about things they particularly liked to do in and out of school. Once they seemed relaxed, she said:

> *"I've spoken to a lot of people and they've said to me that little boys/girls about as old as you don't know much about numbers and worse than that, some of them have said that you don't know anything about numbers. Is that true?" (Ibid: p.86)*

In this way, social interaction and discourse were a central feature of the exchanges between Gay and the children. Additionally, she encouraged the children to reflect on cognitive conflict and to listen and respond to each other, frequently requesting their opinion of something that their partner had said. If she deemed information was necessary for a child to be able to make sense, she provided it without comment.

Children's narratives

From the extensive transcriptions in Gay's thesis, I have chosen to focus on evidence of the children behaving in four ways which contribute to their learning, authoring, sense-making, collaborating, and using non-verbal narratives. The first of these is authoring.

> "As an author, the learner uses his or her mathematical voice to enquire, interrogate and reflect upon what is being learned and how. What does it mean to say that a learner of mathematics is an author? For the majority of classrooms, authorship appears to be vested in the mathematicians who determine what is to be learned, and the texts through which that mathematics is conveyed. We believe that such a view ignores what is known about the process of coming to know which, far from being one of cultural transmission, is necessarily one of interpretation and meaning negotiation in the context of current personal 'knowing' as well as knowledge situated in the community. This we believe to be a lifelong struggle to accord meanings to the narratives that describe the personal, the socio-cultural and, inevitably, the political." (Povey & Burton, 1999: 232)

Authoring

Nathan was counting from 1. He reached 12, and then jumped to 31. Gay pointed out:

> "At this time Nathan was struggling to make sense of teen symbolizations, their linguistically similar decade forms (17 and 70) and reversals (35 and 53). Thus, saying 31 after 12 is significant in the sense that it is the translation for the reversal of the next symbolization" (Ibid: 115).

With this explanation, Nathan's attempts to create a story that made sense to him make it obvious how misguided can be the reading of such counting habits as 'errors'. Nathan continued to count until he reached "thirty ten and thirty twelve", composite terms often employed by children at this stage in their learning as they attempt to make meaning of some of the absurdities of counting in the English system. Having learnt to count from 1 to 10, it seems eminently sensible to extend and use this pattern with numbers higher than this, an example of the use of presupposition.

At the start of the study, Elizabeth knew four of the numbers 1 to 10, did not know any of the numbers higher than 10 and would not make suggestions even with encouragement. Part-way through the study, Elizabeth was struggling to make sense of the relationship between digit position and name particularly with respect to decade names. When looking at the dartboard, she gave 41 as the translation for 14, and called 12, the next symbolization on the dartboard, 42. Later, 17, followed by 19, provoked 71 then 79. Later, Elizabeth was translating teen numbers quite successfully and any mis-takes were to do with decade names (13 called 33), linguistic similarity (14 called 40) or reversed symbolizations (17 called 71). Elizabeth, challenged to read three digit numbers and despite not having been exposed to their additive components, accepted the challenge and referred to 134 as "34 and a 100", a fair and accurate reading which, historically, is possibly nearer correct.

Gay commented:

> "I believe that my research indicates that if traditional assumptions with regard to appropriate learning experiences were ignored and young children were allowed to define the limits of their own learning and participate in the framing of learning experiences, then they might be empowered to come to understand a great deal more about the structure of the number system than they do at present." (p.138)

Later, I will take up the implications for teachers. Now I move to the second aspect.

Sense-making

Elizabeth and Mel were ordering double digit numbers. Gay repeated their findings:

Gay: *The biggest was 55 because if we put them in order it would be 33, then 44, then 55.*
Elizabeth: *Because a 3 goes by a 4 and a 5 goes there.*
And then it's supposed to be a 6 and a 7 and an 8, then a 9, then a 10.
Mel: *They are both countings.*
Elizabeth: *It goes 8, then 9. 8 goes there.*
Mel: *6, 7, 8, 9.*

Here, we see the effectiveness of children working together to construct joint stories but also how far children can exceed adult expectations when challenged. In contrast, Mel was perceived by her teacher as the least 'able' child in the class with respect to number and had commented that Mel seemed "to be going backwards in class...She has already been through the work twice, numbers 2 and 3 and is going through it for the third time and she is still not grasping it. I asked her to draw 3 things this morning and she drew one and that was within 2 minutes of explaining it to her."

Marion Bird in her book *Mathematics for Young Children* (1991) writes about Sam aged 5.03, who made configurations of 5 black dots in a 3 x 3 square. Of the first two in his middle row, he made an unprovoked observation of their symmetry, saying: 'That one's facing that way and that one's facing that way' (p110).
Sam, building upon his interest in visual stories, went on, in a highly systematic way, to make all the available different patterns of 8 dots simply by identifying the positioning of the ninth:
Sam has clearly exhausted all possible positions in a mathematically highly convincing and meaningful way. He authored this himself using a diagrammatic form of story-telling.

The third aspect which affects learning of mathematics is collaborating, of which we have already seen some benefits. There follow further examples.

The benefits of collaboration

Nathan and Stephen were asked what was the biggest number they knew. Nathan suggested 252 thousand million 2000 whereupon Gay asked him to think about what he had just said. He looked thoughtful and then changed his response to 252 thousand million. Stephen then formulated his offering of 252 and 20 million trillion and Nathan responded by saying 252 thousand million trillion. Gay referred to this as 'mirroring' where one child uses some of the other child's response in an imitative way but makes it her/his own by extending it. I think it is noticeable how enjoyable it is to children to play imaginatively with number constructions in this way. It can also be very useful to a teacher in mapping zones because it is indicative of inter-subjectivity.

Oliver and Greg were asked, by Gay, if they could do some adding up sums.

Oliver: *Umm, well, not very hard ones like 100 add a 100.*
Gay: *You couldn't do a 100 add a 100? [Pause 2 seconds.] Umm, well, what if I asked you to do 20 add 1?*
Greg: *That's too hard for me.*
Gay: *What about you Oliver? If you pretend that in your head that you have got 20 and then in your head you put 1 more?*
Oliver: *21!*
Gay: *Yes, you see you can do it. Pretend in your head you have got a 100.*
Oliver: *Arh!*
Gay: *And one more.*
Oliver: *100 and 1!*
Gay: *Pretend in your head you have got a 1000 and 1 more.*
Oliver: *A 1000 and 1.*
Greg: [very quietly] *A 1000 and 1.*
Gay: *OK, pretend in your head you have got a 100 and 10 more.*
Oliver: *A 100 and 10. I just guessed that!*
Gay: *OK, Greg pretend in your head 200 and 1 more comes along.*
Greg: *200 and 1.*
Gay: *Oliver, in your head you have got 200 and 20 comes along.*
[2 seconds pause]
Greg: *I will help you. 200 and tw..., tw..., tw...*
Gay: *Well done, Greg.*
Oliver: *200 and 20.*

Gay used this transcript to exemplify scaffolding at work between Oliver and Greg. She pointed out that their awareness of relationships between numbers was facilitated by the presentation of measure embodiment experiences and that allowed them to begin to make sense of the additive place value component of numbers. Furthermore, she believed that they were coming to understand how to use this knowledge to calculate as, when Oliver paused, it provided some evidence that he was not just repeating the number names he heard but had made a connection which enabled him to work out the answer. However, for me, what is central to this piece of transcript is that the two boys are together building a story about the construction of big numbers, a story which is essential to their ultimate understanding of how place value functions.

Non-verbal narrative

Finally, we move to non-verbal narrative. Katy's partner asked her to make the calculator display 230. The constant had been set to +10. She approached this problem by stopping on 30 and staring at it, then stopping on 130 and commenting that there needed to be a 2 at the front before finally stopping on the correct symbolization. Gay interpreted Katy's actions as reflecting a sub-division approach, which Gay had modeled on numerous occasions, thus enabling Katy to acquire it's use herself. Gay suggested that it can be inferred from this example that it is possible, through her reflection, for a child to learn to be simultaneously the 'more knowledgeable' other, and the learner, thus breaking down the supposed distinction between scaffolding and apprenticeship (see Ibid: p.117).

As teachers, what guidance do children's mathematical narratives give us?

Children's narratives as guidance for teachers

The children's narratives and the sense that the teacher makes of them rely on the assumption that children are always trying to make meaning. This underlines their assumption of both agency and of authoring. Instead of identifying errors, or looking for differences from the mathematics as taught or offered in the text, as teachers and researchers, I believe we have to create the opportunities for the making of narratives and then look inside the children's stories to try and make sense of *their* meanings. Such a process is highly informative as well as being supportive of the children. In the case of a child imposing, on some mathematics, a sense which is inconsistent with convention, I prefer to describe this using the English word 'mis-take' rather than 'error'. There is then nothing to stop new 'takes', which explore the space between imaginative and paradigmatic narrative.

What then are particular features of utilizing children's narratives, which are informative to teachers? I would like to draw attention to four, which I believe to be of great importance. The first is the message provided by instability. The second is teachers deliberately using mis-takes, in this case about counting, as information about children's mathematical development. The third is avoiding the domination of the written and the fourth is inconsistency between knowledge and understanding of structure.

The Message of Instability

Gay worked with Matthew in his first term in school. In the first interview Gay had with Matthew and his partner, Matthew's string from 18 to 28 was correct, except for the omission of 20. His stable correct sequence increased during this first term at school while working with Gay. By the end of that term, however, Matthew omitted 15, 21 to 25 and the whole of the twenties decade appeared to have become unstable. Gay suggested that a reason for this might have been that Matthew's classroom experiences had focused on the numbers 1 to 10 so he had not had opportunities to reflect on the number name sequence beyond this and, as a result, the connections he was beginning to make had not been maintained. If this conjecture is correct, two issues become critical for teachers. One is that children require appropriate presentations to maintain the width of their zone of proximal development. The second is the teacher's need to recognise and nurture reflection as a necessary condition for learning mathematics. When a child appears to regress, the focus of questions must be on the classroom experience and *not* on the child.

Counting mis-takes as information

When children learn to count, they do so by demonstrating an observation of pattern, which, in the case of the English number system, is not always reliable. The counting story that they tell, consequently, is not redolent of error but is informative about their pattern observation. A child who counts to nine and then names ten as onety, for example, following this with onety-one and so on up to twoty, then threety is being entirely consistent with what, conventionally, follows with forty, fifty, and so on. The fact that we do not count in this way in English is a historical accident and a response to other cultural conditions, not an 'objective' reality. There has been extensive research within mathematics education on the many different cultural variations of number naming constructions and the advantages to the learner, over English, of being born into a culture that has a counting style which is consistent and logical.

Nathan, whom we met earlier, was asked to name the numbers in order. He correctly recited 1 to 14 followed by 52, 53, 54 and 55. At the end of that same term, Nathan could only recite correctly to 12 but followed this by a string from 31 to 38, with two omissions, 34 and 35 and then with thirty ten and thirty twelve. This, at first sight, suggested that Nathan's correct sequence had decreased. However, as pointed out earlier, the incorrect sequence may have lain in Nathan's struggle to make sense. Gay claimed that the apparent regression in Nathan's correct sequence might best be interpreted as indicative of cognitive advancement in symbolizations, which would be a help to the teacher in mapping Nathan's zone of proximal development.

Avoiding the domination of the written

The overwhelming message, which reaches children when they start school, is of the importance of the written form. English children frequently complain that the trouble with going on an outing from school is that, afterwards, they will have to write about it!
As Götz Krummheuer (2001) has pointed out, a strength of narrative is that it is oral in form. We tell stories, in the first instance. Subsequently, writing provides an opportunity to reflect again, confirm and substantiate the argument, make 'visible' the ephemeral of the spoken. "The children are supposed to find means of presenting their thoughts which last over a longer span of time" (Ibid: 132) as well as providing the teacher with a more stable format on which to work with the children.

Marion Bird, an acknowledged superb teacher and recorder of working with young children, describes giving Vanessa (5.07), Ben (5.05), Anne-Marie (5.08) and Helen (5.03) each "a small plastic bag containing a set of nine squares of card with dots on them. The squares were jumbled but the children's first reaction was to sort them into groups with the same number of dots on each square, commenting on the numbers of dots as they were doing this" (1991, p.45).
She then gave them some blank 3 x 3 square grids which the children used to record their arrangements and asked them to record how many dots were in each row. Finally, she asked the children to arrange their squares in different ways on fresh grids. Marion Bird recorded the children's comments about the activity:

> "1. Ben was delighted that *each* of the rows of numbers in his first square came to six. He said that they were 'All the same number' and added 'I didn't know I could make a six with a two, a three and a one!' He also commented that he had 'Got two sixes' in his second example…A quick glance at what Ben had done revealed that he had four lots of one dot instead of three. I decided to ask him how many one dots, two dots and three dots he had on his grid. He counted and showed some surprise at there being just two twos but four ones. To *my* surprise he realized immediately that he needed to switch a one dot to a two dot which he then did in the third row. Furthermore, apparently without any further counting, he changed the '6' to a '7'. (Ibid: p.47/48)

"2. Helen had completed her first example as shown. For her second example, she put three, two and one along the top row as shown. When she looked at this afterwards, she was intrigued to find that what she had going across the paper in her first grid, now went down the paper in her second grid! She decided to write her original totals of nine, six, three along the bottom of the second grid. She also totalled the dots going across the bottom row and, having found that this gave six, said that the other rows would be six too. Furthermore, she counted to see how many dots she had on her grid *altogether*. She reached 18 correctly and wrote the numerals for this, also correctly at the side of her grid." (Ibid: p.49).

"3. Vanessa drew the dots on her grids without putting any on the squares of card. When I first saw her grid she had written just one '15' on it, the one by the side of the middle row. I thought that perhaps she had counted the three fours incorrectly but when I asked her to tell me about what she had done her comments revealed that she had counted the number of dots in both the top and middle rows. I expressed some surprise and said that I had thought, wrongly, that the '15' referred to just the middle row. Immediately she wrote on the other '15' and said that this was to help me see what she meant! It is also interesting to see that her dots in the third row total fifteen as well and to ponder whether she made this so on purpose." (Ibid: p.52)

These children were making use of two things that are essential to this kind of classroom. First, they were exploring stories in pictures as well as words, a facility which is profoundly useful in mathematics and often ignored. Second, they were involved in a community of learners challenged in a similar way. The productions of these children, therefore, should not be seen as personal but as the results of interactivity

Another example, provided by Marion Bird, of the productive interactivity of children and their creative response to a puzzling situation follows:

"A group of 4 and 5 year olds had spent a session coloring-in strips of squares of different lengths using red and green pencils alternately. They had become interested in which numbers of squares gave two red ends, two green ends, or one red and one green end. When I brought in the strips the following session for the group to continue working with them, I commented on them being in a muddle in the envelope and invited Xanthe (4.09) and Leanne (5.00) to sort them out in some way. Leanne started to collect the strips with two red ends saying, 'All the reds on this pile'. Xanthe added 'And all the greens on this are coming over here', and started to collect the strips with two green ends. After a while, I noted that Leanne had included in her pile strips with one red and one green end. I asked Xanthe what she thoughts about that and she claimed that it should be on *her* pile. Then suddenly she had another idea. She turned the strip round and placed it so that the red end went on Leanne's pile and the green end went on her own pile. With an air of satisfaction she announced 'Half goes on each end'. (Ibid: p.108/109)

The teacher, on this occasion, encouraged this resolution by creating conditions for learning that depended upon the pupils working together, assuming responsibility for their learning, having their offerings valued and being expected to arrive at, and communicate, a sensible outcome.

Inconsistency between knowledge and understandings of structure: The example of Infinity

Nathan, our young friend already encountered working with Gay, counted from 100 to 103 and then said:

past a thousand, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, it keeps going for all of the numbers of a hundred then trillions.

This was at the same time that Nathan was demonstrating difficulties in the teen sequence and the linguistic and the symbolic similarities were constituting an obstacle to his recitation. Yet despite this, he had knowledge of the relationship between terms much further on in the sequence. He knew that 64 came after 50, 510 came before 990 and 940 came after 870. Gay conjectured that his knowledge of the decade sequence had begun to mature along with his knowledge of the order of name changes and when to make them. Consequently, she claimed that the relative shortness of his number name recitation was misleading with regard to the state of his coming to know. His statement inferring infinity indicates that his understanding of the structure of the numbers was greater than his ability to name a particular sequence, which could be very misleading for his teacher. This is a pointer to the damage that can be done by an over-zealous reliance on the limitations of a text or of textual material, in the absence of the space and time to explore the meanings of the text for the children as well as their linking stories i.e. the stories that one activity provokes them to tell about another.

Conclusion: Trusting children to identify their own understandings

The teacher's comments about Mel, already noted, surprised Gay as Mel was able to make sense of 3 digit symbolizations and displayed great tenacity when she and Elizabeth were faced with a challenge. In her final interview with Mel's teacher, Gay found out that Mel had been referred to the Educational Psychologist for assessment. Gay forwarded her summary statements to him with a covering letter, but received no reply. It would appear that, for this teacher and this Educational Psychologist, an alternative perspective on Mel's performance would have provoked too many questions about what was being offered to Mel in class and how expectations were being laid on Mel which might, ultimately, constrain and distort her approach to learning mathematics.

The kinds of narratives that have been reported here are rich with information for the practicing teacher, if she is open to such observations and encounters with children. In a classroom where such information is valued and encouraged, and where learners are expected to reflect, and make and justify their inferences and deductions, the children themselves freely identify their own understandings.

For example, Alice, Katy and Gay had each thrown three dice and scored 443, 655 and 556 respectively. The biggest number was the winner but Alice had no idea whose number that was. Katy explained:

"*I had a 6, you had a 4, Alice, and Gay had a 5.*"

While Katy was speaking, Alice was looking around the room and Gay was unsure if she had heard what Katy had said so she asked Alice to repeat it. Alice retorted:

"*I know what she said, 6, 4 and 5, but I don't know what she means actually*".

Alice was able to identify what she did not understand because she had been engaged in a programme which encouraged her to reflect and vocalize.

In summary, then, left to their own devices, children are natural story-tellers and they practice this in *all* aspects of their learning, including mathematics. The ways in which we constrain the learning of mathematics, however, distort the potential for the kinds of experiences which can feed their narratives and which can help them to make sense of these narratives as well as shift them towards the narratives which are conventionally recognized. The experience, across many countries, of the dysfunctionality of teaching paradigmatic mathematics in the absence either of the children's need to know, or of connections to their knowing style, should be evidence enough of a pressing requirement for change in classrooms. There is, additionally, evidence of how practicing research mathematicians utilize the very narrative strategies that, here, I am identifying as productive for young children's learning (see, for example, Burton, 1999, Burton, 2001). What a pity that a search for mathematical meaning by young children cannot be as recognized and valued by them, their teachers and their communities, as the published research outcomes of pursuing similar strategies by mathematicians. I believe that the learning of mathematics is locked into a 'normal science' (Kuhn, 1970) paradigm which fails to distinguish between knowing and knowledge, between the construction of narrative and it's formalization. Within such an approach, I believe it is not difficult to comprehend why children find mathematics both unpalatable and unacceptable – in their words, "boring". I have offered an alternative perspective on mathematics, one which is consistent with approaches to the learning of other disciplines but also one which enables the learner to inhabit and make sense of the mathematical world that they are entering. Since such an approach is the one naturally used by mathematicians engaged in attempting to make sense of a research problem when they, themselves, are trying to learn, how appropriate it would be to take these practices as normal and natural to *all* learners of mathematics and treat early, indeed all, learning as a research practice in which story telling, the construction of convincing narratives, is the name of the game.

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